# RAJIV GANDHI UNIVERSITY OF KNOWLEDGE TECHNOLOGIES

SUBJECT: MATHEMATICS　　SEMESTER: II
DATE : 17-04-2010　　MAXIMUM MARKS: 60
TIME : 3 Hrs.

## (SEMESTER EXAM)

*This paper consists of three sections A, B and C. The section A carry 40 marks and the sections B and C carry 20 marks to be answered in an answer booklet supplied to you.*

## Section-A (40 Marks)

## Multiple Choice Questions:

1. The equation of the tangent to the curve $x^2 + y^2 = y^4 - 2x$ at $(-2,1)$ is

A. $x + y + 1 = 0$

B. $x + y - 1 = 0$

C. $x - y + 1 = 0$

D. $x - y - 1 = 0$

ANS: A

2. If $y = (1 - \sqrt{x})^{-1}$ then $\frac{d^2y}{dx^2} =$

A. $\frac{1}{2}(1 - \sqrt{x})^{-2}\left(\frac{3}{2} - \frac{1}{2\sqrt{x}}\right)$

B. $\frac{1}{2}(1 - \sqrt{x})^{-3}\left(\frac{3}{2} - \frac{1}{2\sqrt{x}}\right)$

C. $\frac{1}{2x}(1 - \sqrt{x})^{-2}\left(\frac{3}{2} - \frac{1}{2\sqrt{x}}\right)$

D. $\frac{1}{2x}(1 - \sqrt{x})^{-3}\left(\frac{3}{2} - \frac{1}{2\sqrt{x}}\right)$

ANS: D

3. If $f(\theta) = \left(\frac{sin\theta}{1+cos\theta}\right)^2$ then $f'(\theta) =$

A. $\frac{sin\theta}{(1+cos\theta)^2}$

B. $\frac{2sin\theta}{(1+cos\theta)^2}$

C. $\frac{4sin\theta}{(1+cos\theta)^3}$

D. $\frac{sin\theta}{(1+cos\theta)^3}$

ANS: B

4. The function $f(x) = 2\cos 2x - \cos 4x$ has an absolute minimum in $[0, \pi]$ at $x =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{5\pi}{6}$

D. $\pi$

ANS: B

5. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0,2)$.

A. $\cos x + 3$

B. $-\cos x + 3$

C. $\cos x - 3$

D. $-\cos x - 3$

ANS: B

6. The interval on which the function $f(x) = \sin 2x - x$, $0 < x < \frac{\pi}{2}$ decreases is

A. $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

B. $\left(0, \frac{\pi}{6}\right)$

C. $\left(0, \frac{\pi}{4}\right)$

D. $\left(0, \frac{\pi}{3}\right)$

ANS: A

7. $\lim_{x \to \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}\right] =$

A. 0

B. $\frac{1}{2}$

C. 1

D. $\infty$

ANS: B

8. The curve $y = tan\, x$ has vertical asymptotes at

A. Integer multiples of $\pi$

B. Odd integer multiples of $\frac{\pi}{2}$

C. Even integer multiples of $\frac{\pi}{2}$

D. Odd integer multiples of $\pi$

ANS: B

9. The oblique asymptote of the graph $f(x) = \frac{x^2}{x-1}$ is

A. $y = x - 1$

B. $y = x + 1$

C. $y = -x + 1$

D. $y = -x - 1$

ANS: B

10. The linearization of $f(x) = 2 - \int_{2}^{x+1} \frac{9}{t+1}\, dt$ at $x = 1$ is

A. $L(x) = 2x + 1$

B. $L(x) = 2x - 5$

C. $L(x) = -3x + 5$

D. $L(x) = 3x - 1$

ANS: C

11. If the radius of a circle is increased from $5\ cm$ to $5.06\ cm$, then the estimated percentage change in the area of the circle is

A. 2.2

B. 2.4

C. 2.6

D. 2.8

ANS: B

12. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\, dx = f(x)+C \Rightarrow f(x) =$

A. $\frac{1}{2}\sqrt{1+x}$

B. $\frac{2}{3}(1+x)^{3/2}$

C. $\frac{2}{3}\sqrt{1+x}$

D. $\frac{2}{3}(x+x^2)^{3/2}$

ANS: B

13. $\int \frac{\left(x^3-x\right)^{1/3}}{x^4}\, dx = g(x)+C \Rightarrow g(x) =$

A. $\frac{3}{8}\left(1-\frac{1}{x^2}\right)^{4/3}$

B. $\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3}$

C. $\frac{3}{8}(1-x^2)^{4/3}$

D. $\frac{3}{8}(1-x^3)^{4/3}$

ANS: A

14. Evaluate $\int_{-1}^{1} (2-|x|)\,dx$

A. 1

B. 2

C. 3

D. 4

ANS: C

15. Find the upper bound of the integral $\int_{0}^{1} \sqrt{x+8}\,dx$

A. $2\sqrt{2}$

B. 3

C. 1

D. $-3$

ANS: B

16. Find the area of the region between the graph of the function $f(x) = x^3 - 4x$ and the $x$ −axis over the interval $[-2,2]$.

A. 0

B. 4

C. 8

D. 12

ANS: C

17. If $y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}$, $|x| < \frac{\pi}{2}$ then find $\frac{dy}{dx}$

A. $secx$

B. $cosx$

C. 1

D. 0

ANS: C

18. Evaluate the integral $\int_0^2 \sqrt{4-x^2}\ dx$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\pi$

ANS: D

19. Find the area of the region enclosed by the curves $y = 7 - 2x^2$ and $y = x^2 + 4$.

A. $\frac{4}{3}$

B. $\frac{2}{3}$

C. 4

D. 2

ANS: C

20. Find the length of the curve $x = \int_{-3}^{y} \sqrt{t^4 - 1}\, dt$,

$-3 \leq y \leq 3$

A. 3

B. 9

C. 12

D. 18

ANS: D

21. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$ , $1 \leq x \leq 2$ about the $x$ −axis.

A. $\frac{8\pi}{3}[3\sqrt{3} - 2\sqrt{2}]$
B. $\frac{8\pi}{3}[2\sqrt{2} + 3\sqrt{3}]$
C. $\frac{3\pi}{8}[3\sqrt{3} - 2\sqrt{2}]$
D. $\frac{3\pi}{8}[3\sqrt{3} + 2\sqrt{2}]$

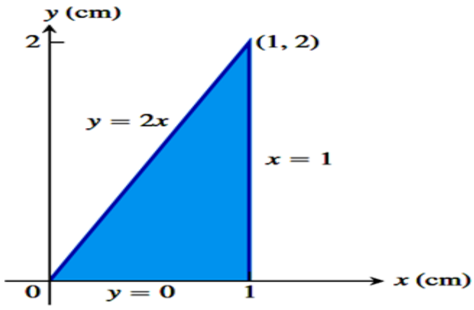
ANS: A

22. If the density function of a thin rod is $\delta(x) = 1 + \frac{1}{\sqrt{x}}$, $1 \leq x \leq 4$ lying along the $x$ −axis, then find the rod's center of mass.

A. $\frac{73}{6}$
B. $\frac{1}{5}$
C. $\frac{73}{5}$
D. $\frac{73}{30}$

ANS: D

23. The triangular plate shown below has a constant density of $\delta = 6\ g/cm^2$ .




Find the plate's moment (in $g.cm$) about the $y$ —axis.

A. 1

B. 2

C. 3

D. 4

ANS: D

24. If the amount of work done to stretch a spring from its natural length of $2\ m$ to a length of $5\ m$ is $1800\ J$ , then find the spring's force constant (in $N/m$).

A. 200

B. 400

C. 600

D. 800

ANS: B

## Matching:

### I. Match questions 25 - 27 with a correct answer from the options given below:

**OPTIONS:**

A. $2$

B. $1$

C. $-1$

D. $-2$

25. If $x^3 + y^3 = 16$ then $\frac{d^2y}{dx^2}$ at the point $(2,2)$

ANS: D

26. The slope of curve $(x^2 + y^2)^2 = (x - y)^2$ at $(1, -1)$ is

ANS: B

27. The slope of the normal to the curve $x^2y^2 = 9$ at $(2,1)$

ANS: A

### II. Match questions 28 - 30 with a correct answer from the options given below:

**OPTIONS:**

A. A point of inflection and the curve is increasing $\forall\, x \in R, x \neq 0$
B. A point of inflection and the curve is decreasing $\forall\, x \in R, x \neq 0$
C. A cusp and a local minimum.
D. A cusp and a local maximum.

28. At $x = 0$, the curve $y = x^{\frac{2}{5}}$ has

ANS: C

29. At $x = 0$, the curve $y = x^{\frac{3}{5}}$ has

ANS: A

30. At $x = 0$, the curve $y = x^{\frac{2}{3}}(x - 5)$ has

ANS: D

## III. Match questions 31 - 33 with a correct answer from the options given below:

**OPTIONS:**

A. 0

B. 2

C. 4

D. 6

31. The average value of the function $f(x) = cosx$ on the interval $[0, 2\pi]$ is

ANS: A

32. The area under the graph of the function $f(x) = sinx$ on the interval $[0, 2\pi]$ is

ANS: C

33. Evaluate $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\, dx$

ANS: B

## IV. Match questions 34 - 36 with a correct answer from the options given below:

**OPTIONS:**

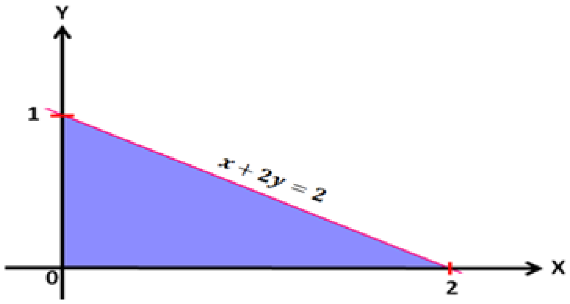
A. $\frac{8\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{2\pi}{3}$

D. $2\pi$

34. Find the volume of the solid generated by revolving the shaded region given below about the $x$ −axis.




ANS: C

35. Find the volume of the solid generated by revolving the region bounded by the curve $y = 2\sqrt{x}$ and the lines $y = 2$, $x = 0$ about the $x$ −axis.

ANS: D

36. Find the volume of the solid generated by revolving the region bounded by the curve $y = |x|$ and $y = 1$ about the $x$ −axis.

ANS: B

## Comprehension:

### I. Read the following passage and answer the questions 37-40.

If $f(x)=\frac{a}{3}x^3+\frac{b}{2}x^2+cx+d$, where $a, b, c, d \in \boldsymbol{R}$, $a \neq 0$ and $b^2-4ac>0$, then

37. The number of critical points of $f(x)$ is

A. $0$

B. $1$

C. $2$

D. $3$

ANS: C

38. The curve $y=f(x)$ has

A. a local maximum, but not a local minimum

B. a local minimum, but not a local maximum

C. both local maximum and local minimum

D. neither a local maximum nor a local minimum

ANS: C

39. If $a < 0$, then the curve $y = f(x)$ is

A. concave up in $\left(-\infty, -\frac{b}{2a}\right)$ and concave down in $\left(-\frac{b}{2a}, \infty\right)$

B. concave down in $\left(-\infty, -\frac{b}{2a}\right)$ and concave up in $\left(-\frac{b}{2a}, \infty\right)$

C. concave up in $\left(-\infty, -\frac{b}{2a}\right) \cup \left(-\frac{b}{2a}, \infty\right)$

D. concave down in $\left(-\infty, -\frac{b}{2a}\right) \cup \left(-\frac{b}{2a}, \infty\right)$

ANS: A

40. The curve $y = f(x)$ has

A. a vertical asymptote at $x = -\frac{b}{2a}$

B. a cusp at $x = -\frac{b}{2a}$

C. absolute maximum at $x = -\frac{b}{2a}$

D. a point of inflection at $x = -\frac{b}{2a}$

ANS: D

## II. Read the following passage and answer the questions 41-44.

**Fundamental Theorem of Calculus part1:** If $f$ is continuous on $[a, b]$ then $F(x) = \int_a^x f(t)\, dt$ is continuous on $[a, b]$ and differentiable on $(a, b)$ and its derivative is $f(x)$;

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\, dt = f(x)$$

41. If $F\left(x^2\right) = \int_1^{x^2} \cos t\, dt$ then find $F'(\pi)$, where prime $(\,'\,)$ denotes the differentiation w.r.t $x$

A. $-2$

B. $-1$

C. $1$

D. $2$

ANS: B

42. If $f(x) = x^2 - \int_2^{x+1} \frac{9}{t+1}\, dt$ then $f''(1) =$

A. 1

B. 2

C. 3

D. 4

ANS: C

43. If $\int_{1}^{x} f(t)\, dt = x^2 - 2x + 1$ then $f'(x) =$

A. 2

B. 4

C. 6

D. 8

ANS: A

44. If $\int_{0}^{x} f(t)\, dt = x\cos \pi x$ then $f(4) =$

A. 0

B. 1

C. 2

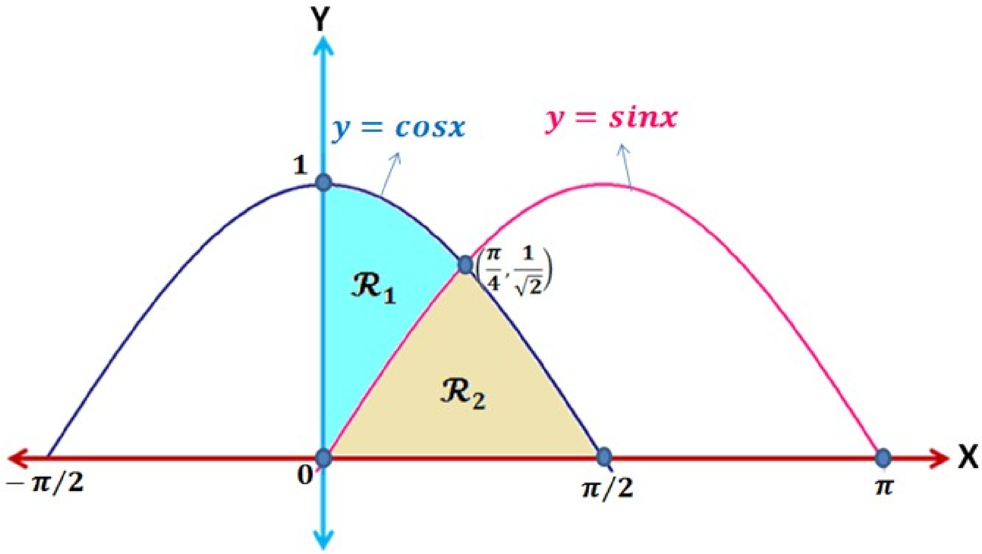
D. 3

ANS: B

## III. Read the following passage and answer the questions 45-48

Observe the following figure carefully and answer the questions given below:

**The curves $y = cosx$ and $y = sinx$**




45. The area of the region $\mathcal{R}_1$ is

A. $2\sqrt{2} - 1$

B. $\sqrt{2} - 1$

C. $\sqrt{2} + 1$

D. $2\sqrt{2} - 1$

ANS: B

46. The volume of the solid generated by revolving the region $\mathcal{R}_1$ about $x$ −axis is

A. $\frac{\pi^2}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\pi$

ANS: C

47. The area of the region $\mathcal{R}_2$ is

A. $2-\sqrt{2}$

B. $2\left(\sqrt{2}+1\right)$

C. $2+\sqrt{2}$

D. $2\sqrt{2}+1$

ANS: A

48. The volume of the solid generated by revolving the region $\mathcal{R}_2$ about $x$ −axis is

A. $\frac{\pi^2}{2}-\frac{\pi}{4}$

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{4}$

D. $\frac{\pi^2}{4}-\frac{\pi}{2}$

ANS: D

## Section: B

**Answer any *two* Questions $(2 \times 5 = 10)$**

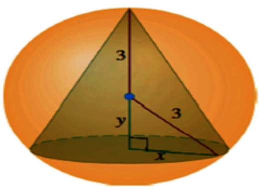
1.

a. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 $km$ north of the intersection and the car is $0.8$ $km$ to the east, the police determine with radar that the distance between them and the car is increasing at $20$ $km/hour$. If the cruiser is moving at 60 $km/hour$ at the instant of measurement, what is the speed of the car?

b. Find the critical points of $f(x) = x^{2/3}(x^2 - 4)$. Identify the intervals on which $f$ is increasing and decreasing. Find the function's local and absolute extreme values.

2.

a. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius $3$.




b. Find the area bounded by the parabola $x^2 = 8y$ , the $x$ —axis and the lines $x = -2$ , $x = 4$.

3.

a. Use the shell method to find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the line $y = x$ about the $y$ —axis.

b. The arc of the parabola $y = x^2$ from $(1,1)$ to $(2,4)$ is rotated about the $y$ —axis. Find the area of the resulting surface.

## Section-C

### Answer any *one* Question $(\mathbf{1} \times \mathbf{10} = \mathbf{10})$

1.

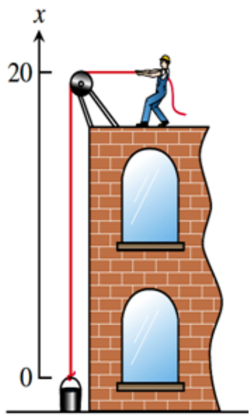
a. State and prove Rolle's Theorem

b. Graph the function $y = \frac{x^3}{3x^2+1}$.

2.

a. State and prove the Fundamental Theorem of Calculus part1.

b. A leaky 5-lb bucket is lifted from the ground into the air by pulling in 20 ft of rope at a constant speed.




The rope weighs 0.08 lb/ft. The bucket starts with 2 gal of water (16 lb) and leaks at a constant rate. It finishes draining just as it reaches the top. How much work was spent in

a) Lifting the water alone
b) Lifting the water and bucket together
c) Lifting the water, bucket, and rope